

# Vidyasagar University



Post Graduate Syllabus  
in  
***Applied Mathematics with Oceanology and  
Computer Programming***

under Choice Based Credit System  
(CBCS)

[ w.e.f. : 2016-2017 ]

**Vidyasagar University**  
***Applied Mathematics with Oceanology and Computer Programming***  
***Midnapore-721 102, West Bengal***  
***Syllabus of M.Sc.***

***With effect from 2016-2017***

***Semester-I***

<i>Course No.</i>	<i>Topics</i>	<i>Marks</i>	<i>No. of Lectures Hours</i>
MTM101	Real Analysis	50	40
MTM102	Complex Analysis	50	40
MTM103	Ordinary Differential Equations and Special Functions	50	40
MTM104	Advanced Programming in C and MATLAB	50	40
MTM105	Classical Mechanics and Non – linear Dynamics	50	40
MTM106	Graph Theory	25	20
Unit-1			
MTM106	Lab. 1: (Computational	25	20
Unit-2	Methods: Using MATLAB)		

***Semester-II***

<i>Course No.</i>	<i>Topics</i>	<i>Marks</i>	<i>No. of Lectures Hours</i>
MTM201	Fluid Mechanics	50	40
MTM202	Numerical Analysis	50	40
MTM203	Advanced Abstract Algebra	25	20
Unit-1			
MTM203	Advanced Linear Algebra	25	20
Unit-2			
MTM204	General Theory of Continuum Mechanics	50	40
MTM205	General Topology	25	20
Unit-1			
MTM205	Lab. 2: (Language: C-	25	20
Unit-2	Programming with Numerical Methods)		
MTM206*	Discrete Mathematics	50	40

### ***Semester-III***

<i>Course No.</i>	<i>Topics</i>	<i>Marks</i>	<i>No. of Lectures Hours</i>
MTM301	Partial Differential Equations and Generalized Functions	50	40
MTM302	Transforms and Integral Equations	50	40
MTM303	Operations Research /Dynamical Oceanology and Meteorology	50	40
MTM304	Special Paper-OM Dynamical Oceanology-I	50	40
MTM305	Special Paper-OM Dynamical Meteorology -I	50	40
MTM304	Special Paper-OR Advanced Optimization and Operations Research	50	40
MTM305	Special Paper-OR Operational Research Modelling-I	50	40
MTM306*	Numerical Methods and Computer Programming	50	40

### ***Semester-IV***

<i>Course No.</i>	<i>Topics</i>	<i>Marks</i>	<i>No. of Lectures Hours</i>
MTM401	Functional Analysis	50	40
MTM402	Fuzzy Sets and their Applications	25	20
MTM402	Stochastic Process and Regression	25	20
MTM403	Magneto Hydro-Dynamics	25	20
MTM403	Soft Computing	25	20
MTM404	Special Paper-OM Dynamical Oceanology -II	50	40
MTM405	Special Paper-OM Dynamical Meteorology -II	25	20
MTM405	Lab.: (Dynamical Meteorology)	25	20
MTM404	Special Paper-OR Nonlinear Optimization	50	40
MTM405	Special Paper-OR Operational Research Modelling-II	25	20

MTM405	Lab. on Special Paper: 25	20
Unit-2	(Operations Research: Using MATLAB/LINGO/ MATHEMATICA)	
MTM406	Dissertation Project Work	50

**Note:**

There will be two examinations for each paper: (i) End semester examination having 40 marks and (ii) Internal assessment (IA) examination having 10 marks. Marks from IA will be evaluated by averaging two marks obtained in two IA examinations. In this course there are two special papers: Dynamical Oceanology and Meteorology (MTM 304, 305, 404 and 405) and Operations Research (MTM 304, 305, 404 and 405). Each student must take any one of these two.

\*Open elective papers (for all PG students other than students of Applied Mathematics.)

## ***Semester-I***

### **MTM101      Real Analysis**

**50**

Complete Metric spaces, compactness, connectedness(with emphasis on  $\mathbb{R}^n$ ), Heine-Borel Theorem, Separable and non-separable metric spaces.

Functions of bounded variation, R-S Integral

Measurable sets. Concept of Lebesgue function. Inner and outer measure. Its simple properties. Set of measure zero. Cantor set, Borel set and their measurability, Non-measurable sets.

Measurable function: Definition and it's simple properties, Borel measurable functions, sequence of measurable functions, Statement of Lusin's theorem, Egoroff's theorem. Simple functions and it's properties.

Lebesgue integral on a measurable set: Definition. Basic simple properties.

Lebesgue integral of a bounded function over a set A of finite measure. Simple properties. Integral of non-negative measurable functions, General Lebesgue integral. Bounded convergence theorem for a sequence of Lebesgue integrable function, Fatou's lemma. Classical Lebesgue dominated convergence theorem. Monotone convergence theorem, Relation between Lebesgue integral and Riemann integral

### **MTM102      Complex Analysis**

**50**

Complex numbers. The complex plane. Functions of a complex variable. Limit. Continuity. Differentiability. The definition of an analytic function. Cauchy- Riemann differential equation. Construction of analytic function.

Complex integration. Jordan arc. Contour. Rectifiable arcs. The absolute value of complex integral. Cauchy's theorem. Cauchy's integral formula. The derivatives of an analytic function. Cauchy's inequality. Morera's theorem. Liouville's theorem. Taylor's and Laurent's series. Maximum modulus principle.

Singularities : Zero of an analytic function. Different types of singularities. Poles. Isolated. Removal and Essential singularities.

Residues: Residue at pole. Residue at infinity. Cauchy residue theorem. Number of poles and zeros of an analytic function. Rouché's theorem.

Contour integration: Evaluation of integrals using contour integration.

Conformal representation: Conformal transformation. Möbius transformation or Bilinear transformation. Mapping properties of important functions.

### **MTM103      Ordinary Differential Equations and Special Functions**

**50**

**Differential equation.** Homogeneous linear differential equations. Fundamental system of integrals. Singularity of a linear differential equation. Solution in the neighbourhood of a singularity. Regular integral. equation of Fuchsian type. Series solution by Frobenius method.

**Hypergeometric equation.** Hypergeometric functions. Series solution near zero. one and infinity. Integral formula for the hypergeometric function. Differentiation of hypergeometric function. The confluent hypergeometric function. Integral representation of confluent hypergeometric function.

**Legendre equation:** Legendre functions, Generating function, Legendre functions of the first kind and second kind, Laplace integral, Orthogonal properties of Legendre polynomials, Rodrigue's formula, Schlaefli's integral.

**Bessel equation:** Bessel function, Series solution of Bessel equation, Generating function, Integrals representations of Bessel's functions, Hankel functions, Recurrence relations, Asymptotic expansion of Bessel functions.

**Green's Function:** Green's Function and its properties, Green's function for ordinary differential equations, Application to Boundary Value Problems.

**Eigen Value Problem:** Ordinary differential equations of the Sturm Liouville type, Properties of Sturm Liouville type, Application to Boundary Value Problems. Eigenvalues and eigenfunctions, Orthogonality theorem, Expansion theorem.

**System of Linear Differential Equations:** Systems of First order equations and the Matrix form, Representation of nth order equations as a system, Existence and uniqueness of solutions of system of equations, Wronskian of vector functions.

**MTM104      Advanced Programming in C and      50**  
**MATLAB**

**Programming in C:** Review of basic concepts of C programming, Arrays, structure and union, Enum, pointers, pointers and functions, pointers and arrays, array of pointers, pointers and structures, strings and string handling functions, Dynamic memory allocation: using of malloc(), realloc(), calloc() and free(), file handling functions: use of fopen, fclose, fputc, fgetc, fputs, fscanf, fprintf, fseek, putc, getc, putw, getw, append, low level programming and C preprocessor: Directive, #define, Macro Substitution, conditional compilation, #if, #ifdef, #ifndef, #else, #endif.

**Programming in MATLAB:** The Matlab workspace, data types, variables, assignment statements, arrays, sets, matrices, string, time, date, cell arrays and structures, introduction to M – file scripts, input and output functions, conditional control statements, loop control statements, break, continue and return statements.

**MTM105      Classical Mechanics and Non – linear      50**  
**Dynamics**

Motion of a system of particles. Constraints. Generalized coordinates. Holonomic and non-holonomic system. Principle of virtual work. D'Alembert's Principle. Lagrange's equations. Plane pendulum and spherical pendulum. Cyclic co-ordinates. Coriolis force. Motion relative to rotating earth.

Principle of stationary action. Hamilton's principle. Deduction of Lagrange from Hamilton's principle. Brachistochrone problem. Lagrange's equations from Hamilton's principle.. Invariance transformations. Conservation laws. Noether's theorem. Infinitesimal transformations. Space-time transformations. Legendre transformation. Hamiltonian. Hamilton's equations. Poisson bracket. Canonical transformations. Liouville's theorem.

Hamilton-Jacobi equation for Hamilton's principle function. Solution of harmonic oscillator problem by Hamilton-Jacobi method.

Small oscillation about equilibrium. Lagrange's method. Normal co-ordinates. Oscillations under constraint. Stationary character of a normal mode. Small oscillation about the state of steady motion. Normal coordinates

Orientation and displacement of a rigid body. Eulerian angles. Cayley-Klein parameters. Inertia tensor. Eigenvalues of the inertia tensor. Principal axis transformation. Euler equations of motion. Motion of a free body about a fixed point.

Special theory of relativity in Classical Mechanics:-Postulates of special relativity. Lorentz transformation. Consequences of Lorentz transformation. Force and energy equations in relativistic mechanics.

Nonlinear Dynamics: Linear systems. Phase portraits: qualitative behavior. Linearization at a fixed point. Fixed points. Stability aspects. Lyapunov functions (stability theorem). Typical examples. Limit cycles. Poincare-Bendixson theory. Bifurcations. Different types of bifurcations.

**MTM106      Graph Theory      25**  
**Unit-1**

Elements of graph theory. Eulerian and Hamiltonian Graphs. Trees. Planar graphs. Distance and centre. Duals. Cut sets and cut vertices. Bipartite graphs. Colouring and matching. Four colour theorem (statement only). Planar graphs. Directed graphs and weighted graphs. Matrix representation of graphs. Intersection graph, Applications of graphs in operations research.

**MTM106      Lab.: (Computational Method: Using MATLAB)      25**  
**Unit-2**

Problem: 20 marks; Lab. Note Book and Viva-Voce: 5.

1. **Working with matrix:** Generating matrix, Concatenation, Deleting rows and columns. Symmetric matrix, matrix multiplication, Test the matrix for singularity, magic matrix. Matrix analysis using function: norm, normest, rank, det, trace, null, orth, rref, subspace, inv, expm, logm, sqrtm, funm.
2. **Array:** Addition, Subtraction, Element-by-element multiplication, Element-by-element division, Element-by-element left division, Element-by-element power. Multidimensional Arrays, Cell Arrays, Characters and Text in array,
3. **Graph Plotting:** Plotting Process, Creating a Graph, Graph Components, Figure Tools, Arranging Graphs Within a Figure, Choosing a Type of Graph to Plot, Editing Plots, Plotting Two Variables with Plotting Tools, Changing the Appearance of Lines and Markers, Adding More Data to the Graph, Changing the Type of Graph, Modifying the Graph Data Source, Annotating Graphs for Presentation, Exporting the Graph.
4. **Using Basic Plotting Functions:** Creating a Plot, Plotting Multiple Data Sets in One Graph, Specifying Line Styles and Colors, Plotting Lines and Markers, Graphing Imaginary and Complex Data, Adding Plots to an Existing Graph, Figure Windows, Displaying Multiple Plots in One Figure, Controlling the Axes, Adding Axis Labels and Titles, Saving Figures.
5. **Programming:** Conditional Control – if, else, switch, Loop Control – for, while, continue, break, Error Control – try, catch, Program Termination – return.
6. **Scripts and Functions:** Scripts, Functions, Types of Functions, Global Variables, Passing String Arguments to Functions, The eval Function, Function Handles, Function Functions, Vectorization, Preallocation.
7. **Data Analysis:** (i) Preprocessing Data : Loading the Data, Missing Data, Outliers, Smoothing and Filtering, (ii) Summarizing Data: Measures of Location, Measures of

Scale, Shape of a Distribution, (iii) Visualizing Data: 2-D Scatter Plots, 3-D Scatter Plots, Scatter Plot Arrays, Exploring Data in Graphs, (iv) Modeling Data: Polynomial Regression, General Linear Regression,

8. **Linear Algebra:** Systems of Linear Equations, Inverses and Determinants, Factorizations, Powers and Exponentials, Eigenvalues, Singular Values.
9. **Polynomials:** Polynomial functions in the MATLAB® environment, Representing Polynomials, Evaluating Polynomials, Roots, Derivatives, Convolution, Partial Fraction Expansions, Polynomial Curve Fitting, Characteristic Polynomials.



**Irrotational Motion in Two Dimensions:** General motion of a cylinder in two dimensions. Motion of a cylinder in a uniform stream, Liquid streaming past a fixed circular cylinder and two coaxial cylinders. Equations of motion of a circular cylinder. Circulation about a moving cylinder. Conjugate function. Elliptic cylinder. Liquid streaming past a fixed elliptic cylinder. Elliptic cylinder rotating in an infinite mass of liquid at rest at infinity. Circulation about an elliptic cylinder. Kinetic energy. Blasius theorem and its application. Kutta and Joukowski theorem, D'Alembert's paradox. Application of conformal mapping.

**Vortex Motion:** Vortex line, Vortex tube, Properties of the vortex, Strength of the vortex, Rectilinear vortices, Velocity component, centre of vortices. A case of two vortex filaments, vortex pair. Vortex doublet. Image of vortex filament with respect to a plane. An infinite single row of parallel rectilinear vortices of same strength. Two infinite row of parallel rectilinear vortices, Karman's vortex street. Rectilinear vortex with circular section. Rankine's combine vortex. Rectilinear vortices with elliptic section.

**Viscous Flow:** Navier-Stokes equations, Vorticity and circulation in viscous fluids. Reynolds number, Boundary conditions, Flow of a viscous fluid with free surface on an inclined plane. Flow between parallel plates. Flow through pipes of circular, elliptic section under constant pressure gradient. Laminar flow between concentric rotating cylinder. Steady motion of a viscous fluid due to a slowly rotating sphere. Unsteady motion of a flat plate. Pulsatile flow between parallel surfaces. Prandtl's concept of boundary layer. Boundary layer flow along a flat plate. Momentum and energy integral equation for the boundary layer. Von Karman Pohlhausen method. Turbulence, Calculation of Turbulent BL.

**MTM202      Numerical Analysis****50**

Symbolic operators and their relations.

Cubic spline interpolation. Lagrange's bivariate interpolation. Approximation of function. Chebyshev polynomial: Minimax property. Curve fitting by least square method. Use of orthogonal polynomials. Economization of power series.

Numerical integration: Newton-Cotes formulae-open type. Gaussian quadrature: Gauss-Legendre, Gauss-Chebyshev. Integration by Monte Carlo method.

Roots of polynomial equation: Bairstow method. Solution of a system of non-linear equations by fixed point method and Newton-Raphson methods. Convergence and rate of convergence.

Solution of a system of linear equations: Matrix inverse. LU decomposition method. Solution of tri-diagonal system of equations. Ill-conditioned linear systems. Relaxation method.

Eigenvalue problem. Power method. Jacobi's method.

Solution of ordinary differential equation: Runge-Kutta method to solve a system of equations and second order IVP. Predictor-corrector method: Milne's method. Stability. Solution of second order boundary value problem by finite difference and finite element methods.

Partial differential equation: Finite difference scheme. Parabolic equation: Crank-Nicolson method. Iteration method to solve Elliptic and hyperbolic equations.

**MTM203      Advanced Abstract Algebra**  
**Unit-1**

**25**

**Groups:** Morphism of groups. Quotient groups. Fundamental theorem on homomorphism of groups. Isomorphism theorems. Automorphism. Solvable groups and theorems on them. Direct product. Conjugacy. Conjugate classes. Class equation. Theorems on finite groups. Cauchy's theorem. Sylow's theorem. Application of Sylow's theorem, Simple groups, Permutation groups, Cayley theorem, Group actions.

**Rings and Field:** Integral domain. Fields. Skew fields. Quotient rings. Morphism of rings. Ideals (Prime and maximal). Isomorphism theorem. Euclidean domain. Principal ideal domain. Unique factorization domain. Polynomial rings.

**MTM203      Advanced Linear Algebra**  
**Unit-2**

**25**

**Linear Transformation:** The algebra of linear transformations, Kernel, Range, Sylvester law, Rank and Nullity, Invertibility and Isomorphism, Matrix representation of a linear transformation, Linear operator, Linear functional, Dual space,

**Minimal Polynomial :** Minimal Polynomial of a Linear Operator, Diagonalization of a Linear Operator, Jordan Canonical Forms.

**Lattice:** Partially and totally ordered sets, Lattice, Semi Lattice, Complete Lattice, Sub lattice, Modular and Distributive Lattices, Complements, Boolean Lattice.

**MTM204      General Theory of Continuum**  
**Mechanics**

**50**

**Stress:** Body force. Surface forces. Cauchy's stress principle. Stress vector. State of stress at a point. Stress tensor. The stress vector –stress tensor relationship. Force and moment equilibrium. Stress tensor symmetry stress quadric of Cauchy. Stress transformation laws. Principal stress. Stress invariant. Stress ellipsoid.

**Strain:** Deformation Gradients. Displacement Gradient Deformation tensor. Finite strain tensors. Small deformation theory-infinitesimal strain tensor. Relative displacement. Linear rotation tensor. Interpretation of the linear strain tensors. Strength ratio. Finite strain interpretation. Principal strains. Strain invariant. Cubical dilatation .

Compatibility equation for linear strain. Strain energy function. Hook's law. Saint –Venant's principal. Airy's strain function. Isotropic media. Elastic constraints. Moduli of elasticity of isotropic bodies and their relation. Displacement equation of motion. Waves in isotropic elastic media.

**Perfect fluid:** Kinematics of fluid. Lagrangian method.. Eulerian method. Acceleration . Equation of continuity. The boundary surface.. Stream lines and path lines. Irrotational motion and its physical interpretation. Velocity potential. Euler's equation of motion of an in viscid fluid. Cauchy's integral. Bernoulli's equation. Integration of Euler's equation.

Impulsive motion of fluid. Energy equation. Motion in two dimensions. The stream functions Complex potential. Source, sink and doublet and their images. Milne-Thompson circle theorem and its application. Vorticity. Flow and circulation. Kelvin's circulation theorem. Kelvin's minimum energy theorem.

Topological Spaces: open sets, closed sets, neighbourhoods, bases, subbases, limit points, closures, interiors, continuous functions, homeomorphisms. 1<sup>st</sup> and 2<sup>nd</sup> countable spaces. Examples of topological spaces: subspace topology, product topology, metric topology, order topology, Quotient Topology.

Connectedness and Compactness: Connected spaces, Connected subspaces of the real line, Components and local connectedness, Compact spaces, Local –compactness, Tychonoff's Theorem on compact spaces

Separation Axioms: Hausdorff spaces, Regularity, Complete Regularity, Normality, Urysohn Lemma, Urysohn Metrization Theorem, Tietze Extension theorem.

**MTM205      Lab.: (Language: C Programming with**  
**Unit-2      Numerical and Statistical Methods)**

**25**

Problem: 20 marks; Lab. note book and viva: 5.

The programs are to be written on the following problems using pointers, data file, structures, etc.

**On Searching and Sorting Problems:** Linear and binary search, Bubble, Insertion, Selection techniques.

**String manipulation:** No of occurrence of a letter in a given string, Palindrome nature of string, Rewrite the name with surname first, Print a string in a reverse order, String searching, Sorting of names in alphabetic order, Find and replace a given letter or word in a given string, Combinations of letters of a word, Conversion of name into abbreviation form, Pattern matching

**On Numerical Problems:**

- (i) Evaluation of determinant by Gauss elimination method, using partial pivoting.
- (ii) Matrix inverse by partial pivoting.
- (iii) Roots of Polynomial equation.
- (iv) Solution of system of linear equations by Gauss Seidal iteration method, Matrix inversion method, LU decomposition method, Gauss elimination method.
- (v) Solution of Tri-diagonal equations.
- (vi) Interpolation: Difference table, Lagrange, Newton forward and backward interpolation, Cubic spline interpolation.
- (vii) Integration: Gauss quadrature rule, Integration by Monte Carlo method, Double integration.
- (viii) Solution of ODE: Eulers and Modified Eulers, Runge-Kuta, Predictor and Corrector method: Milne method.
- (ix) Solution of PDE by Finite difference method.
- (x) Eigen value of a matrix: Power method, Jacobi method.

**On Statistical Problems:**

- (i) On bivariate distribution: Correlation coefficient, Regression lines, Curve fitting.
- (ii) Multiple regression.
- (iii) Simple hypothesis testing.

**Boolean algebra:** Introduction, Basic Definitions, Duality, Basic Theorems, Boolean algebra and lattice, Representation Theorem, Sum-of-product form for sets, Sum-of-products form for Boolean Algebra. Propositional Logic, Tautology

**Sets and propositions:** Cardinality. Mathematical Induction. Principle of Inclusion and exclusion.

**Computability and Formal Languages:** Ordered Sets. Languages. Phrase Structure Grammars. Types of Grammars and Languages.

**Finite State Machines:** Equivalent Machines. Finite State Machines as Language Recognizers.

**Partial Order Relations and Lattices:** Chains and Antichains.

**Graph Theory:** Definition, walks, paths, connected graphs, regular and bipartite graphs, cycles and circuits. Tree and rooted tree. Spanning trees. Eccentricity of a vertex radius and diameter of a graph. Centre(s) of a tree. Hamiltonian and Eulerian graphs, Planar graphs.

**Analysis of Algorithms:** Time Complexity. Complexity of Problems. Discrete Numeric Functions and Generating Functions.

## ***Semester-III***

### **MTM301      Partial Differential Equations and Generalized Functions**

**50**

#### **Partial Differential Equations:**

First order PDE in two independent variables and the Cauchy problem. Semi-linear and quasi-linear equations in two-independent variables.

Second order linear PDE. Adjoint and self-adjoint equations. Reduction to canonical forms. Classifications. Fundamental equations: Laplace, Wave and Diffusion equations.

#### **Hyperbolic equations:**

Equation of vibration of a string. Existence, uniqueness and continuous dependence of the solution on the initial conditions. Method of separation of variables. D'Alembert's solution for the vibration of an infinite string. Domain of dependence. Riemann's method of solution. Riemann-Volterra solution. Higher-dimensional wave equations.

#### **Elliptic equations:**

Fundamental solution of Laplace's equations in two and three independent variables. Harmonic function. characterization of harmonic function by their mean value property. Uniqueness. Continuous dependence and existence of solutions. Method of separation of variables for the solutions of Laplace's equations. Dirichlet's and Neumann's problems. Green's functions for the Laplace's equations in two and three dimensions. Solution of Dirichlet's and Neumann's problem for some typical problems like a disc and a sphere. Potentials due to a volume distribution, a single layer and a double layer. Representation of a Harmonic function by potentials of simple and double layers. Poisson's general solution.

#### **Parabolic equations:**

Heat equation - Heat conduction problem for an infinite rod - Heat conduction in a finite rod - existence and uniqueness of the solution.

Generalized Functions : Dirac delta function and delta sequences. Test functions. Linear functionals. Regular and singular distributions. Sokhotski-Plemelj formulas. Operations on distributions. Derivatives. Transformation properties of delta function. Fourier transform of generalized functions.

### **MTM302      Transforms and Integral Equations**

**50**

**Fourier Transform:** Properties of Fourier transform, Inversion formula, Convolution, Parseval's relation, Multiple Fourier transform, Bessel's inequality, Application of transform to Heat, Wave and Laplace equations.

**Laplace Transform:** Laplace transform, Properties of Laplace transform, Inversion formula of Laplace transform(Bromwich formula), Convolution theorem, Application to ordinary and partial differential equations.

**Wavelet Transform:** Time-frequency analysis. Multi-resolution analysis. Spline wavelets. Sealing function. Short-time Fourier transform. Wavelet series. Orthogonal wavelets. Applications to signal and image processing.

**Integral Equation:** Formulation of integral equations, Integral equations of Fredholm and Volterra type, solution by successive substitutions and successive approximations. Resolvent

Kernel Method. Integral equations with degenerate kernels. Abel's integral equation, Integral equations of convolution type and their solutions by Laplace transform, Fredholm's theorems. Integral equations with symmetric kernel, Eigenvalue and eigenfunction of integral equation and their simple properties. Fredholm alternative.

**MTM303      Operations Research /Dynamical Oceanography and  
Meteorology**

**50**

**Either**

**(For the student whose special paper is OM)**

**Elements of Optimization and Operations Research. (OR)**

**Revised simplex method** (with and without artificial variables).

**Post optimality analysis** : Change in the objective function, change in the requirement vector, addition of a variable, addition of a constraint.

**Classical optimization techniques** : Single variable optimization, multivariate optimization (with no constraint, with equality constraints and with inequality constraints).

**Integer programming** : Gomory's cutting plane algorithm, (Gomory's mixed integer program algorithm) A branch and bound algorithm,

**Optimal Control** : Methods of calculus of variations, simple optimal problems of mechanics. Inventory model (deterministic).

**Non-linear Programming:** Quadratic Programming : Wolf's modified simplex method and Beale's method.

**Convex programming.** Dynamic programming. Decomposition principle due to Dantzig and Wolfe.

**Or**

**(For the student whose special paper is OR)**

**Dynamical Oceanology and Meteorology (OM)**

**Dynamical Oceanology:** Navier-Stokes equations of motion for viscous fluid. Thermodynamics of sea-water in equilibrium state. Salinity. Basic thermodynamics. Gibb's general thermodynamics relation for sea-water. Governing equations of motion of sea water. Boundary conditions at the free ocean surface. Linearised equation of small amplitude oceanian wave motion on a rotating earth. Boussinesq approximation. The beta plane approximation, Equation of conservation of energy for linearised wave motion.

**Dynamical Meteorology:** Heat balance of the atmosphere, Basic thermodynamics of the atmosphere. Potential temperature and stability of dry air.

Energy in a compressible atmosphere, change in potential energy due to adiabatic interchange of small parcels, dissipation of energy.

General circulation, its schematic description and theory (in out line).

Rate of change of circulation. Geostrophic and thermal wind. The geostrophic balance, The geodynamical paradox. Surface of discontinuity. Classification of fronts. Formation of cyclones. Aerological diagrams, its purpose and use.

**MTM304      Special Paper-OM  
Dynamical Oceanology-I**

**50**

**Thermodynamics of Equilibrium State:** Gibb's relation, Thermodynamic potentials. Definition of Salinity, Sea water as two component mixture, Entropy, Internal energy and Chemical potential of sea water, Adiabatic gradient of temperature and coefficient of compressibility of sea water, equilibrium conditions of sea water.

**Thermodynamics of Irreversible Processes :** Fluxes of heat and salt, conservation equations for heat, salt and mass, Navier-Stokes equations in an inertial frame and the corresponding equation in a uniformly rotating frame. Potential Vorticity. The- plane approximation. Boussinesq approximation,

Basic concept of turbulence, Reynold's stresses, equation of turbulent energy, Coefficient of turbulent exchange. Closer of the system of average equation for the large scale flow, Boundary conditions, Ageostrophic motion, Ekman layer on a free surface, Vertical Shear layers.

**MTM305      Special Paper-OM  
Dynamical Meteorology-I**

**50**

**Thermodynamics of Atmosphere:** Basic laws of thermodynamics, potential temperature, Lapserate. Atmospheric water vapour. Equation of state for Dry and Moist air, Dry weather, virtual Temperature, Adiabatic changes in unsaturated Moist Air and in saturated Moist Air, Pseudoadiabatic change, Equivalent Temperature and Equivalent Potential Temperature, Saturated adiabatic Lapse Rate, Stability in saturated adiabatic changes, conditional Instability, Isobaric Cooling, Condensations by Isobaric cooling.

**Aerological Diagrams:** Purpose and use of Aerological Diagrams, Clapeyron and Refsdal and Stuve diagrams, Tephigram, Emagram , Area equivalence.

**Dynamics in Atmosphere:** Compositions and structure of the Atmosphere. Solar and Terrestrial radiations. Derivation of the complete of thermo-Hydrodynamical equation w.r.t. the Rotating Earth, Energy and Angular Momentum, Circulation and vorticity, Rate of change of circulation, The geostrophic gradient, Thermal and cyclostrophic winds, The geodynamical Paradox, Steady motion along a circular Isobar, accelerated motion and a changing pressure field, Divergence, Convergence and pressure variation, Pressure distribution in a moving cyclone, Pressure-tendency equation, The atmospheric energy equation, The dissipation of energy, Available potential energy.

**MTM304      Special Paper-OR  
Advanced Optimization and Operations Research**

**50**

**Revised simplex method** (with and without artificial variable). Modified dual simplex .

**Large Scale Linear Programming: Decomposition Principle of Dantzig and Wolf.**

**Parametric and post-optimal analysis:** Change in the objective function. Change in the requirement vector, Addition of a variable, Addition of a constraint, Parametric analysis of cost and requirement vector.

**Classical Optimization Techniques:** Multivariate optimization (with no constraint, with equality constraints and with inequality constraints)

**Search Methods:** Fibonacci and golden section method.



**Gradient Method:** Method of conjugate directions for quadratic function, Steepest descent and Davodon-Fletcher-Powell method.

Methods of feasible direction and cutting hyperplane method.

**Integer Programming:** Gomory's cutting plane algorithm, Gomory's mixed integer problem algorithm, A branch and bound algorithm.

**Goal Programming:** Introduction, Difference between LP and GP approach, Concept of Goal Programming, Graphical solution-method of Goal Programming, Modified simplex method of Goal Programming.

Optimization for Several Variables: Algebraic approach, Algebraic geometrical approach, cost – different approach, Inequality approach

**MTM305      Special Paper-OR  
Operational Research Modelling-I**

**50**

**Dynamic Programming:** Introduction, Nature of dynamic programming, Deterministic processes, Non-Sequential discrete optimization, Allocation problems, Assortment problems, Sequential discrete optimization, Long-term planning problem, Multi-stage decision process, Application of Dynamic Programming in production scheduling and routing problems.

**Inventory control :** Inventory control -Deterministic including price breaks and Multi-item with constraints , -Probabilistic (with and without lead time), Fuzzy and Dynamic inventory models. Basic concept of supply – chain management and two echelon supply chain model.

**Queuing Theory :** Basic Structures of queuing models, Poisson queues –M/M/1, M/M/C for finite and infinite queue length, Non-Poisson queue -M/G/1, Machine-Maintenance (steady state).

**Network : PERT and CPM:** Introduction, Basic difference between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM Network components and precedence relationships, Critical path analysis, Probability in PERT analysis, Project Time-Cost, Trade-off, Updating of the project, Resource allocation — resource smoothing and resource leveling.

**Replacement and Maintenance Models:** Introduction, Failure Mechanism of items, Replacement of items deteriorates with time, Replacement policy for equipments when value of money changes with constant rate during the period, Replacement of items that fail completely— individual replacement policy and group replacement policy, Other replacement problems — staffing problem, equipment renewal problem.

**Simulation:** Introduction, Steps of simulation process, Advantages and disadvantages of simulation, Stochastic simulation and random numbers— Monte Carlo simulation, Random number, Generation, Simulation of Inventory Problems, Simulation of Queuing problems, Role of computers in Simulation, Applications of Simulations.



**Numerical methods:**

Sources and causes of errors. Types of errors. Lagrange's and Newton's interpolation. Roots of algebraic and transcendental equations: Bisection, fixed point iteration, Newton-Raphson methods. Rate of convergence. Solution of system of linear equations: Gauss-elimination method, Gauss-Seidal method. Integration by trapezoidal and Simpson 1/3 methods. Solution of ordinary differential equation by Euler's method, Runge-Kutta methods. Second order IVP and BVP.

**Computer programming in C:**

Input and output statements. If, goto, statements. Arrays. Loops. Recursion. Structure. Pointer. Use of files. Macros. Programming of above mentioned numerical problems.

## ***Semester-IV***

### **MTM401      Functional Analysis**

**50**

Normed spaces. Continuity of linear maps. Bounded linear transformation. Set of all bounded linear transformation  $B(X,Y)$  from NLS  $X$  into NLS  $Y$  is a NLS.  $B(X,Y)$  is a Banach space if  $Y$  is a Banach space. Quotient of normed linear spaces and its consequences. Hahn-Banach Extension theorem and Its applications. Banach spaces. A NLS is Banach iff every absolutely convergent series is convergent. Conjugate spaces, Reflexive spaces. Uniform Boundedness Principle and its applications. Closed Graph Theorem, Open Mapping Theorem and their applications. Inner product spaces, Hilbert spaces. Orthonormal basis. Complete Orthonormal basis. Cauchy-Schwarz inequality. Parallelogram law. Projection theorem. Inner product is a continuous operator. Relation between IPS and NLS. Bessel's inequality. Parseval's identity. Strong and Weak convergence of sequence of operators. Reflexivity of Hilbert space. Riesz Representation Theorem for bounded linear functional on a Hilbert space. Definition of self adjoint operator, Normal, Unitary and Positive operators, Related simple theorems.

### **MTM402      Fuzzy Sets and Their Applications** **Unit-1**

**25**

Definition of Fuzzy sets. Alpha-set. Normality Extension Principle. Basic Operations like inclusion. Completion, Union and intersection, Difference.

Fuzzy numbers. Addition, Subtraction, Multiplication and Division, Triangular and trapezoid fuzzy numbers.

Linear Programming Problems with fuzzy resources :

(i) Vendegay's approach

(ii) Werner's approach

L.P.P. with fuzzy resources and objective : Zimmermann's approach.

L.P.P. with fuzzy parameters in the objective function. Definition of Fuzzy multiobjective linear programming problems. A brief survey of the methodology of solving fuzzy M.O.L.P. and fuzzy goal programming.

### **MTM402      Stochastic Processes and Regression** **Unit-2**

**25**

Stochastic Process: Markov chains with finite and countable state space. Classification of states. Limiting behavior of  $n$  state transition probabilities. Stationary distribution. Branching process. Random walk. Gambler's ruin problem. Markov processes in continuous time. Poisson's process. Partial correlation. Multiple correlation. Advanced theory of linear estimation.

<b>MTM403</b>	<b>Magneto Hydro-Dynamics</b>	<b>25</b>
<b>Unit-1</b>		

Maxwell's electromagnetic field equations when medium in motion. Lorentz's force. The equations of motion of a conducting fluid. Basic equations. Simplification of the electromagnetic field equation. Magnetic Reynolds number. Alfven theorem. Magnetic body force. Ferraro's law of isorotation. Laminar Flow of a viscous conducting liquid between parallel walls in transverse magnetic fields. M.H.D. Flow Past a porous flat plate without induced magnetic field. MHD Couette Flow under different boundary conditions, Magneto hydro dynamics waves. Hall currents. MHD flow past a porous flat plate without induced magnetic field.

<b>MTM403</b>	<b>Soft Computing</b>	<b>25</b>
<b>Unit-2</b>		

Introduction of soft computing, fuzzy logic, Genetic Algorithm, Neural networks, Application of fuzzy logic concepts in scientific problems, Solution of optimization problems using Genetic Algorithm. Neural Network approaches in scientific analysis, design, and diagnostic problems.

<b>MTM404</b>	<b>Special Paper-OM Dynamical Oceanology-II</b>	<b>50</b>
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The vorticity equation, Potential vorticity, The thermal wind, The Taylor-Proudman theorem, inertial waves, Geostrophic motion, Consequences of the Geostrophic and Hydrostatic. Rossby number, Ekman number, The shallow-water equations, Potential vorticity conservation. Plane wave in a layer of constant depth. Poincare-Kelvin waves. The Rossby waves, Dynamics diagnosis of the Rossby waves.

**Homogeneous Models of the wind-driven steady oceanic circulation :** Derivation of the wind-driven steady oceanic circulation, Derivation of the Vorticity equation in terms of geostrophic stream function and the relevant boundary conditions, The Sverdrup relation, Meridional boundary layers, Bottom friction layer, Inertial boundary layer theory, Inertial currents in the presence of friction, dissipation integrals for steady circulation, Ekman upwelling circulation.

**Quasigeostrophic motion of a stratified fluid on a sphere :** The equations of motion in a spherical coordinates, Scaling, geostrophic approximation for synoptic scales, The concept of static stability, quasigeostrophic potential vorticity equation for oceanic synoptic scales and relevant boundary conditions, Rossby Wave normal modes, The vertical structure equation, Topographic waves in a stratified ocean, Geostrophic approximation for large scale, The thermocline problems, Barotropic and Baroclinic Rossby waves.

Internal waves in a rotating nondissipative stratified fluid, Inertial waves.

<b>MTM405</b>	<b>Special Paper-OM Unit-1 Dynamical Meteorology-II</b>	<b>25</b>
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**Fronts, Cyclones and Anticyclones :** Surface of discontinuity, Pressure distribution near Fronts, Temperature and wind distribution at Fronts, Classification of Fronts, The geostrophic Fronts, The motion of Fronts,.

**Global Circulation:** The general circulation and monsoon meteorology Atmospheric wave, Numerical wave prediction, Tropical Cyclones, CISK, The wave theory of cyclones, Storm Surges

**Atmosphere Turbulence :** Prandtl's theory of momentum transfer, The vertical variation of the wind in and above the surface layer, Vertical Mixing, Air Masses.

**MTM405                      Lab. : (Lab. on Special Paper-OM)**  
**Unit-2**

**25**

***Problems on Meteorology***

1. Surface temperature, pressure, humidity, Wind speed and direction measurements.
2. Rainfall and rain measurements.
3. TD charts-analysis.
4. T-  $\Phi$  diagram :
  - i) Geopotential height by isotherm / adiabatic method.
  - ii) To find dry bulb and wet bulb temperature, potential, virtual, equivalent potential, dew point temperatures and mixing ratio.
5. Numerical method and computer techniques related to Meteorological problems, Handling and analysis of Meteorological data.
6. Field worked (5-marks) (compulsory): Students should go to one of the University/Institute/Organization laboratory (preferably in the laboratory of I.M.D, I.I.Sc, I.I.M, N.P.L, I.I.T. etc.) to understand experimental set-ups in advance meteorology (such as Annular experiment for existence of general circulation and Rossby wave, experiment for demonstrating Helmholtz instability, Aerosol measurements, Facsimile recorder for receiving weather charts etc.)

**MTM404                      Special Paper-OR**  
**Non-Linear Optimization**

**50**

**Optimization :** The nature of optimization and scope of the theory. The optimality criterion of Linear programming. An application of Farka's Theorem. Existence Theorem for Linear systems. Theorems of the alternatives.

Optimality in the absence of differentiability, Slater's constraint qualification, Karlin's constraint qualification, Kuhn-Tucker's saddle point necessary optimality Theorem.

Optimality criterion with differentiability and Convexity, Kuhn-Tucker's Sufficient optimality theorem, Duality in non-linear programming, Weak duality theorem, Wolfe's duality theorem, Duality for quadratic programming.

**Quadratic Programming:** Wolfe's modified simplex method, Beale's method. Convex Programming.

**Stochastic Programming :** Chance Constraint programming technique.

**Geometric Programming :** Geometric programming (both unconstrained and constrained) with positive and negative degree of difficulty..

**Games:** Preliminary concept of Continuous Game, Bimatrix Games, Nash Equilibrium, Solution of bimatrix games through Quadratic Programming ( Relation with Nonlinear Programming).

**Multi – objective Non – linear Programming:** Introductory concept and Solution procedure.

**Optimal Control:** Performance indices, Methods of calculus of variations, Transversality Conditions, Simple optimal problems of mechanics, Pontryagin's principle (with proof assuming smooth condition), Linear regulator, Application of dynamic programming in proving Pontryagin's principle, Bang –bang Controls.

**Sequencing:** Problems with n jobs two machines, n-jobs three machines and n-jobs, m-machines.

**Reliability:** Concept, Reliability Definition, System Reliability, System Failure rate, Reliability of the Systems connected in Series or / and parallel. MTBF, MTTF, optimization using reliability, reliability and quality control comparison, reduction of life cycle with reliability, maintainability, availability, Effect of age, stress, and mission time on reliability.

**Information Theory:** Introduction, Communication Processes— memory less channel, the channel matrix, Probability relation in a channel, noiseless channel.

A Measure of information- Properties of Entropy function, Measure of Other information quantities — marginal and joint entropies, conditional entropies, expected mutual information, Axiom for an Entropy function, properties of Entropy function.

Channel capacity, efficiency and redundancy.

Encoding — Objectives of Encoding.

Shannon — Fano Encoding Procedure, Necessary and sufficient Condition for Noiseless Encoding.

**MTM405      Lab. : (Lab. on Special Paper-OR)**  
**Unit-2**

**25**

***Problems on Advanced Optimization and Operations Research are to be solved by using MATLAB / LINGO / MATHEMATICA (20 Marks)***

1. Problems on LPP by simplex method,
2. Problems on LPP by revised simplex method,
3. Problems on stochastic programming,
4. Problems on geometric programming,
5. Problems on bi-matrix games,
6. Problems on queuing theory,
7. Problems on QPP by Wolfe's modified method and Beale's method,
8. Problems on IPP by Gomory's cutting plane method,
9. Problems on Networking (PERT and CPM),
11. Problems on inventory,
12. Problems on Monte Carlo simulation techniques,
13. Problems on dynamic programming.

**Field Work (5 Marks)**

Application for Optimization in real life problem by visiting any Industry/Institute/University/Reputed Institution to understand the practical use of the optimization and making Lab Note Book on the experience gathered during the visit.

**MTM406      Dissertation Project Work      50**

Dissertation Project will be performed on Tutorial/ Review Work on Research Papers. For Project Work one class will be held in every week. Marks are divided as the following: Project Work-25, Presentation-15, and Viva-voce-10. Project Work of each student will be evaluated by the concerned internal teacher / supervisor and one External Examiner. The external examiner must be present in the day of evaluation.